Modified RWG Basis Functions for Analysis of Periodic Structures

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Abstract—**Modifications to the popular Rao-Wilton-Glisson (RWG) triangle subdomain basis functions are described that allow their use in analyzing periodic structures where the electric or magnetic surface current must be allowed to flow across unit cell boundaries. Examples of frequency selective surfaces with this requirement include strip grids and meanderline polarizers.**

> Any approach that cannot handle overlapping elements of arbitrary shape has a serious flaw.
>
> Ben A. Munk [1]

I. Introduction

The triangle subdomain basis functions introduced by Rao, Wilton, and Glisson in 1982 [2] are probably the most popular choice when using the method of moments to solve surface integral equations for arbitrarily shaped objects. Several authors have discussed their use when analyzing frequency selective surfaces (FSSs) [3], [4]. For the most part there is no mention of the issues that arise when attempting to model structures for which currents flow through unit cell boundaries. Apparently the previous researchers have restricted consideration to elements with metallization patterns that lie completely within the unit cell. However, the capability to model extended metallization is required for certain FSS geometries such as infinite strip grids and meanderline polarizers, and also to model currents on bodies of revolution. In [5] the correct behavior was enforced for currents on a body of revolution by copying a portion of the triangulation along the cut line of the developed geometric figure. Here we take a different approach. We extend the definition of the basis functions to include not only adjacent face-pairs of the triangulated surface, but also nonadjacent pairs, each member triangle of which contains an edge lying on the boundary of the unit cell. We show below that enforcing the quasi-periodicity condition on the surface currents requires incorporating a phase shift factor into the definition of the basis functions. With these modifications their application to periodic structures is both natural and efficient.

II. Basis Function Formulation

A. Theory

This discussion will be limited to planar frequency selective surfaces that are periodic in two spatial directions. The application of these ideas to bodies of revolution or other structures periodic in only one dimension is straightforward.

We consider a periodic structure that is invariant to translations of the form $\boldsymbol{r} \rightarrow \boldsymbol{r} + m\boldsymbol{s}_1 + n\boldsymbol{s}_2$, where m and n are arbitrary integers and $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ are the *direct lattice vectors*, a pair of real vectors satisfying

$$\boldsymbol{s}_1 \cdot \hat{\boldsymbol{z}} = \boldsymbol{s}_2 \cdot \hat{\boldsymbol{z}} = 0, \quad \boldsymbol{s}_1 \times \boldsymbol{s}_2 = A > 0. \tag{1}$$

Due to the assumed quasi-periodic nature of the excitation, all fields and currents satisfy the Floquet boundary condition

$$V(\boldsymbol{r}+m\boldsymbol{s}_1+n\boldsymbol{s}_2) = V(\boldsymbol{r})e^{-j(m\psi_1+n\psi_2)}, \quad \forall m, n \in \mathbb{Z} \tag{2}$$

where ψ_1 and ψ_2, the *incremental phase shifts*, are a pair of real numbers determined by the nature of the excitation.

Construction of the basis functions begins with a triangulation of the metallization pattern (for electric surface currents) or its complementary area (for magnetic surface currents) within the *unit cell* (the parallelogram defined by $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$) of the structure. In [2] a basis function is defined over each pair of triangles which share a common edge. In this work, we include not only these adjacent pairs of triangles, but also those pairs of triangles which would be adjacent if one of the pair were translated by $\boldsymbol{s}_1$ or $\boldsymbol{s}_2$ from its actual position. Each member of this pair thus contains an edge lying on the unit cell boundary.

Consider first a typical pair of adjacent triangles; their common edge is not on the boundary of the unit cell. Following [2], Fig. 1 shows two such triangles, T_m^+ and T_m^-, which comprise the support of the mth basis function and which share an interior edge of the triangulated surface. Points in T_m^+ may be designated by either the position vector $\boldsymbol{r}$ which locates them with respect to the global origin, or by $\boldsymbol{\rho}_m^+$, which is defined with respect to the free vertex of T_m^+. The vector $\boldsymbol{\rho}_m^-$ is defined similarly for points in T_m^-, except that it is directed *towards* the free vertex of T_m^-. The basis function associated with the mth edge is then defined as

$$f_m(\boldsymbol{r}) = \begin{cases} \dfrac{e^{j\theta_m^+}}{2A_m^+}\boldsymbol{\rho}_m^+ & \text{if } \boldsymbol{r} \in T_m^+ \\ \dfrac{e^{j\theta_m^-}}{2A_m^-}\boldsymbol{\rho}_m^- & \text{if } \boldsymbol{r} \in T_m^- \\ \boldsymbol{0} & \text{otherwise,} \end{cases} \tag{3}$$

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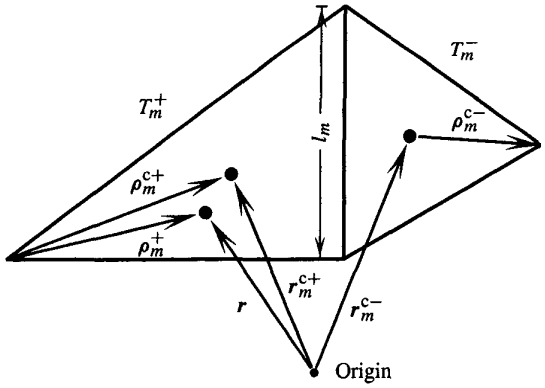

Fig. 1. Triangular basis function geometry showing two triangles with a common edge. The superscript "c" denotes the centroid of the triangle (after [2]).

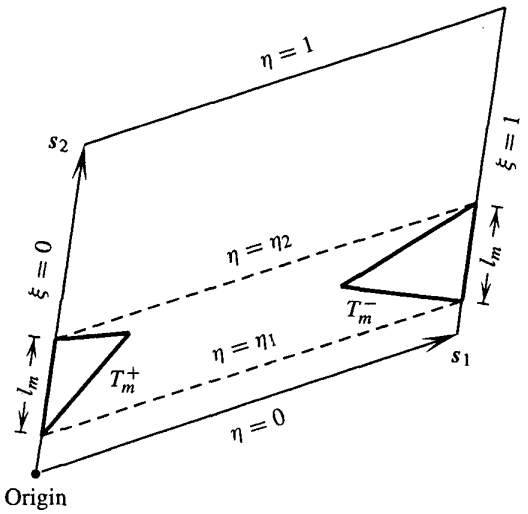

Fig. 2. Triangular basis function geometry showing a pair of triangles located at the ξ = constant unit cell boundaries that occupy the same range of η.

where $A_m^{\pm}$ is the area of triangle $T_m^{\pm}$, and $\theta_m^+ = \theta_m^- = 0$ in this case. Note that in contrast to [2], we have not included the common edge length l_m as a factor in our definition. This choice leads to a useful physical interpretation of the moment method formalism. It implies that the unknown coefficients in the basis function expansion of the electric (magnetic) surface current carry units of current (voltage) and that the generalized impedance (admittance) matrix elements carry units of impedance (admittance). In fact, the expansion coefficient I_m associated with the mth electric current basis function in this case may be interpreted as the total surface current crossing the defining edge. Similarly, the expansion coefficient V_m for the mth magnetic current basis function is total voltage drop across the associated edge.

Apart from the edge length, this definition is similar to that of [2] except for the introduction of the factors containing $\theta_m^{\pm}$. To see why these are necessary, consider the situation shown in Fig. 2. Points (x, y) within the unit cell are parameterized using *unit cell coordinates* ξ and η as follows:

$$\hat{x}x + \hat{y}y = \xi s_1 + \eta s_2, \quad 0 \le \xi < 1, \ 0 \le \eta < 1. \quad (4)$$

Unit cell boundaries are located at $\xi = 0$, $\xi = 1$, $\eta = 0$, and $\eta = 1$. In order to preserve the periodicity of the computed currents we agree to triangulate the unit cell in such a way that the number and location of the resulting edges along the $\xi = 0$ and $\xi = 1$ boundaries are identical, and similarly for the $\eta = 0$ and $\eta = 1$ boundaries. A pair of triangles T_m^+ and T_m^- with edges at the $\xi = 0$ and $\xi = 1$ boundaries, respectively, are shown in the figure. These edges both span the same range of η and so are parallel and congruent. A basis function is defined for each such pair of triangles on the ξ = constant or η = constant boundaries. We focus attention upon the basis function $\boldsymbol{f}_m$ whose support is the union of the two triangle faces shown in the figure. Because of the boundary condition (2) enforced on all currents and fields in the unit cell, it must be true that the normal current density crossing the edge at $\xi = 1$ is equal to that crossing the $\xi = 0$ edge multiplied by the factor $e^{-j\psi_1}$. Therefore, we must insist that

$$\theta_m^- = \theta_m^+ - \psi_1. \quad (5)$$

We will establish the convention that $\theta_m^{\pm} = 0$ for all edges except those along the $\xi = 1$ and $\eta = 1$ unit cell boundaries. Therefore, for the situation shown in Fig. 2 we have $\theta_m^- = -\psi_1$ and similarly for all other triangles with one edge located on the $\xi = 1$ boundary. For triangles with an edge on the $\eta = 1$ boundary we set the corresponding phase to $-\psi_2$.

B. Implementation

The definitions for the modified RWG basis functions have been incorporated into a Fortran 90 program named "PSS" (for polarization selective surface) that uses the mixed potential Green's function formulation of [6].

A number of data structures are used in the program to facilitate calculations involving the basis functions. First, there are the standard matrices used to describe the triangulation: the node list, edge/vertex list, face/vertex list, and face/edge list. In addition, the following matrices are defined to specify the basis functions:

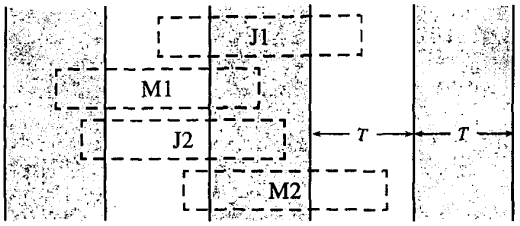

Fig. 3. A section of the symmetric strip geometry showing four possible choices for the rectangular unit cell. The only requirement for the rectangle is that it must be of width $2T$. Metallized regions are shaded.

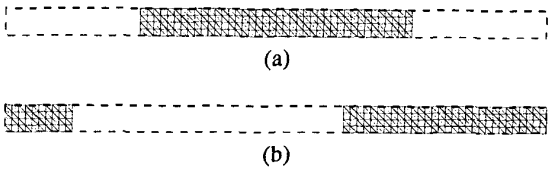

Fig. 4. Triangulations used for the (a) J1 and M1 unit cells, and (b) J2 and M2 unit cells. The dimensions of the unit cells are $P_x = 2T$, $P_y = T/10$.

BFE Basis function/edge list. Entries BFE$(1, m)$ and BFE$(2, m)$ contain respectively the indices of the defining edges on the "plus" and "minus" triangles associated with basis function m. In most cases these two edge indices are identical.

BFF Basis function face list. BFF$(1, m)$ and BFF$(2, m)$ contain respectively the index of the "plus" and "minus" triangle faces associated with basis function m.

EBF Edge/basis function map. EBF(m), if nonzero, is the index of the basis function associated with edge m. Note that several different locations in EBF may be assigned the same value.

ECI Edge cell index, which takes one of the values $\{0, 1, 2, 3, 4\}$ with corresponding meanings (0) The edge does not lie on a unit cell boundary; (1) The edge lies on the $\xi = 0$ boundary; (2) The edge lies on the $\xi = 1$ boundary; (3) The edge lies on the $\eta = 0$ boundary; (4) The edge lies on the $\eta = 1$ boundary.

These arrays are grouped together in a derived type named RWGDATA and are dynamically allocated as needed for a given structure. Note that the correct value of $e^{j\theta_m^{\pm}}$ is easily obtained for edge n by indexing into the five-element vector $[1\ 1\ e^{-j\psi_1}\ 1\ e^{-j\psi_2}]$ using $1 + \text{ECI}(n)$ as the index.

III. Numerical Results

As an example of the use of the modified basis functions, consider the symmetric strip grid shown in Fig. 3, for which an exact series solution is available in [7]. The unit cell is chosen as the rectangle defined by the lattice vectors $s_1 = \hat{x}2T$ and $s_2 = \hat{y}P_y$. Because the structure is invariant in the y direction, P_y can be chosen arbitrarily. The location of the origin is also arbitrary. Rectangles J1 and M1 shown in the figure are customary selections if one chooses to analyze the structure using electric (J) or magnetic (M) surface currents, respectively. Rectangles J2 and M2 are nonstandard choices, since their vertical edges do not coincide with symmetry planes. Nevertheless, by enforcing proper periodicity of the currents using our modified basis functions, these choices are as valid as the others. In fact, it is possible to use electric or magnetic currents for any of the four choices of unit cell. Here, though, we triangulate the metallization (shaded regions) in unit cells J1 and J2 of Fig. 3, which are identical to the nonshaded regions of unit cells M1 and M2. The two triangulations are shown in Fig. 4. Each consists of 320 triangles obtained by adding diagonals to $40 \times 4 = 160$ congruent rectangles. In both parts (a) and (b) of the figure basis functions are defined for the 40 pairs of corresponding boundary triangles adjacent to the top and bottom edges of the unit cell. For the J2 and M2 triangulation (part (b) of the figure), 4 additional basis functions are also defined for corresponding triangle pairs adjacent to the left and right edges of the unit cell.

Calculated reflection coefficient magnitudes and phases from PSS using either electric (J1 and J2) or magnetic (M1 and M2) currents for the two triangulation schemes of Fig. 4 are compared to the exact results from [7] in Figs. 5–8. The calculated results for J1 and J2 are identical, as are those for M1 and M2. This should be expected since the two sets of triangulations are actually identical except for a translation of some of the triangles by s_1. All four calculations agree extremely well with each other and with the exact results.

IV. Conclusions

Modifications have been incorporated into the RWG triangle basis functions that allow them to model periodic structures where electric or magnetic currents flow through unit cell boundaries. The modifications include (i) defining basis functions for corresponding triangle pairs adjacent to opposing unit cell boundaries, and (ii) addition of a phase shift factor to account for the Floquet boundary condition. Data structures needed to easily incorporate these new basis functions into working computer codes have been described. Numerical experiments show that very accurate results are achieved using the new basis functions regardless of the location chosen for the unit cell within the periodic

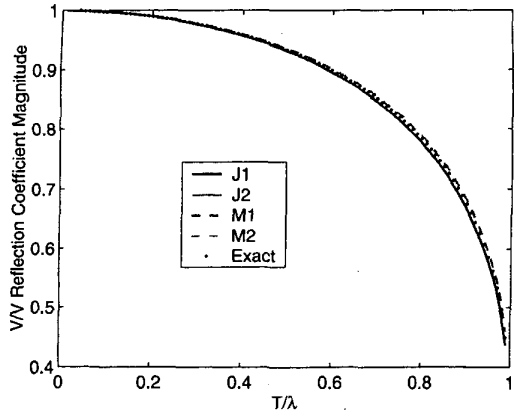

Fig. 5. Reflection magnitude for vertical polarization.

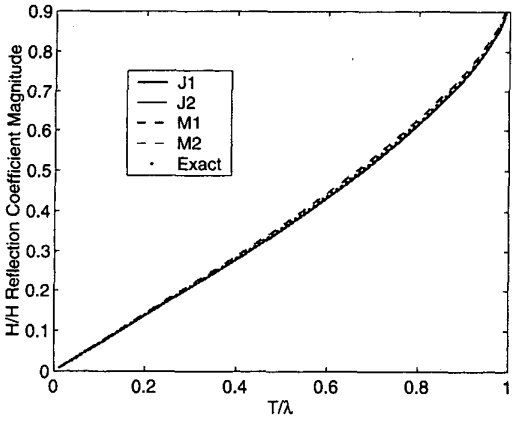

Fig. 6. Reflection magnitude for horizontal polarization.

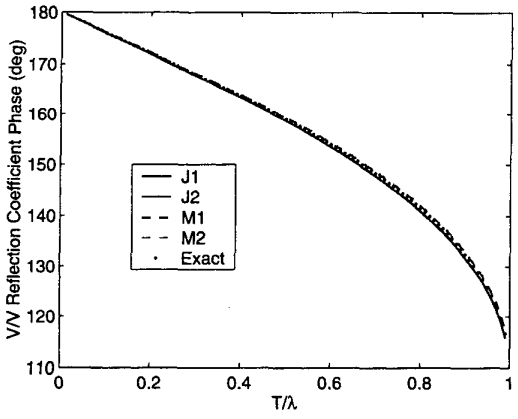

Fig. 7. Reflection phase for vertical polarization.

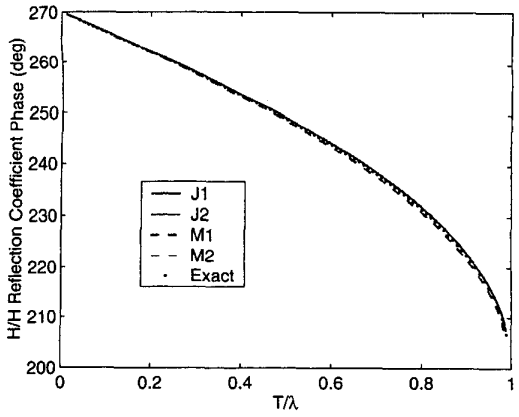

Fig. 8. Reflection phase for horizontal polarization.

structure. The basis functions have been validated for use with both electric and magnetic surface currents.

A unit cell can always be selected for any periodic structure. In fact, the location of the unit cell is completely arbitrary, although certain choices may be more convenient than others. The use of the modified RWG basis functions described here allows one to exploit this freedom. Combined with the inherent versatility of the original RWG basis functions, incorporation of the modified basis functions into the PSS code provides it with the capability to model virtually any type of planar periodic structure.

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